ON THE THERMAL CONTACT RESISTANCE UNDER VACUUM

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The author analyzes the heat exchange under vacuum between two bodies whose adjoining surfaces are in contact at discrete spots uniformly spaced along parallel and equidistant zones. The theoretical values of the thermal contact resistance are compared with experimental data.

The correct calculation of temperature fields in the design of high-temperature apparatus requires a sufficiently accurate determination of the thermal contact resistance between parts which are to be joined together. Formulas for the heat exchange at a contact have been derived in [2] on the basis of theoretical analysis and generalized experimental data.

The thermal contact resistance depends on the microgeometry of the surfaces and the physical characteristics of the surface layer and of the contact medium. The geometry of contact elements in a real situation is determined basically by the surface treatment. In many types of treatment these elements assume the form of ellipses with varying eccentricities [6], but in some cases one may consider the contact between two bodies to be effected along parallel zones spaced with a certain pitch.

In this article we will analyze the steady-state temperature field of a contact uniformly spaced in a plane as shown in Fig. 1. It is assumed that the contact between bodies 1 and 2 is effected along infinite parallel zones of width 2a spaced at a step 2b, and that the heat transfer from one body to the other takes place only by way of thermal conduction through the elements of actual contact. This pattern is approximated by a contact between a surface with regular roughness in one direction and another smooth surface. For most types of surface treatment, according to [6], the angle α is less than 10° and it will be assumed, therefore, that $\alpha = 0$.

Let us consider a steady heat flux in the X direction. Since the contact pitch has been assumed uniform, then in order to determine the temperature field we will select one element contact (Fig. 1b) whose surfaces Y = 0 and Y = b are adiabatic, i.e.,

$$\left.\frac{\partial t}{\partial Y}\right|_{Y=0} = 0 \text{ and } \left.\frac{\partial t}{\partial Y}\right|_{Y=b} = 0 .$$

The angle α will be assumed equal zero.

With a known heat flux q, contact temperature t_C, and thermal conductivity λ_M of the material, the problem reduces to that of solving the Laplace equation

$$\frac{\partial^2 t}{\partial X^2} + \frac{\partial^2 t}{\partial Y^2} = 0 \tag{1}$$

for the following boundary conditions:

1) at X = 0 and Y > b−a, $t = t_C$;

2) at Y = 0, $\partial t/\partial Y = 0$;

3) at Y = b, $\partial t/\partial Y = 0$;

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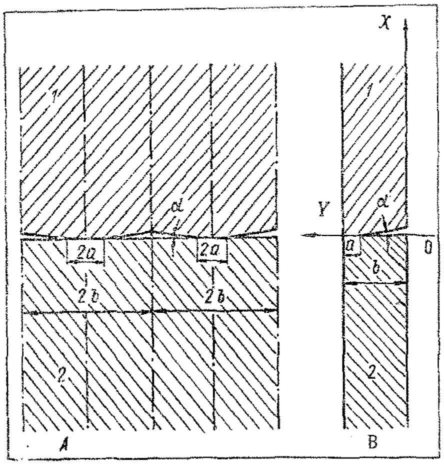

Fig. 1. Two-dimensional contact pattern with a uniform pitch between a rough and a smooth surface: A) idealized contact pattern; B) contact element; 1 and 2 are the bodies in contact, a and b are the contact parameters.

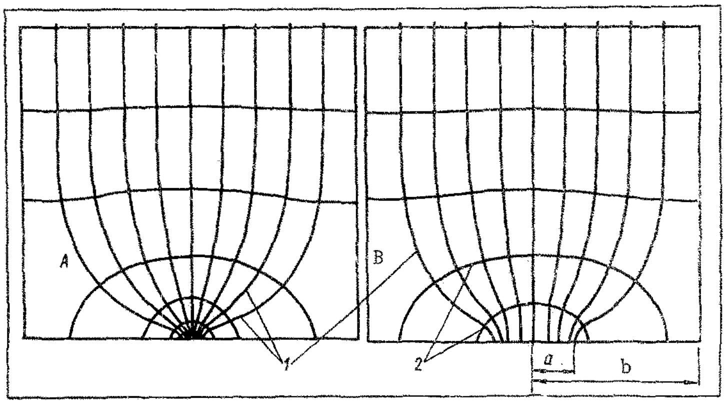

Fig. 2. Heat flow lines and isotherms in the contact zone: A) $\eta = a/b = 0.05$, B) $\eta = a/b = 0.25$; 1) are the thermal current lines; 2) are the isotherms. The temperature gradient between adjacent isotherms is $\Delta u = 0.5$.

4) at $X = \pm\infty$, $\partial t/\partial X = q/\lambda_M$;

5) at $X = 0$ and $Y < b-a$, $\partial t/\partial X = 0$.

The thermal contact resistance represents an additional resistance of the thermal path over and above that of an ideal contact, and it is defined as follows:

$$R_C = \frac{t(\infty) - t(-\infty) - (t_1(\infty) - t_1(-\infty))}{q}, \tag{2}$$

where $t(\infty)$, $t_1(\infty)$, $t(-\infty)$, $t_1(-\infty)$ are the respective temperatures at $X = \infty$ and $X = -\infty$; the subscript 1 refers to an ideal contact. We now introduce the following dimensionless parameters:

$$x = \frac{X}{b}; \quad y = \frac{Y}{b}; \quad \eta = \frac{a}{b}; \quad h = 1 - \eta;$$
$$u = \frac{t - t_K}{bq}\lambda_M; \text{ and } r_C = R_C\frac{\lambda_M}{b}. \tag{3}$$

It is well known from the theory of analytic functions of a complex variable that the real and the imaginary components of such a function satisfy the Laplace equation, the two components being orthogonal functions. In the steady-state temperature field problem, therefore, one component of the analytic function may correspond to equipotential lines (isotherms) while the other component may correspond to force lines of the potential flow (thermal current lines).

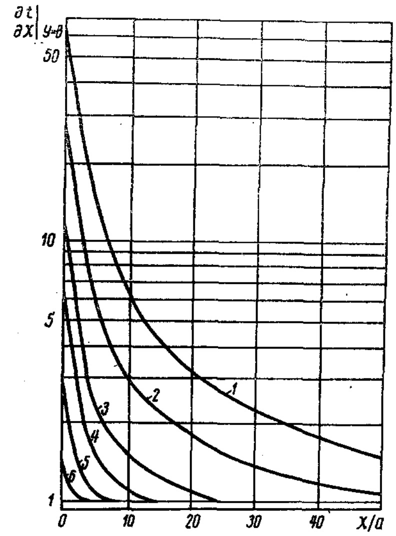

Fig. 3. Temperature gradient $\partial t/\partial X|_{Y=b}$ as a function of the dimensionless distance parameter X/a, when $q/\lambda_M = 1$: 1) $\eta = 10^{-2}$; 2) $2\cdot10^{-2}$; 3) $5\cdot10^{-2}$; 4) 10^{-1}; 5) 0.2; 6) 0.5.

The unknown temperature field is determined by conformal mapping of the region $0 < y < 1$ in the complex plane $z = x + iy$ with a notch ($0 \le y < h$, $x = 0$) into the region $0 < v < 1$ in the complex plane $w = u + iv$. After performing several elementary transformations [1], we obtain the following expression for the complex potential w:

$$w = \frac{2}{\pi}\,\mathrm{ar\,th}\left(\cos\frac{\pi h}{2}\sqrt{\mathrm{th}^2\frac{\pi z}{2}+\mathrm{tg}^2\frac{\pi h}{2}}\right). \tag{4}$$

Separating the real and the imaginary part in expression (8) after a few transformations will give the following equations which relate functions u and v with the coordinates x and y:

$$\frac{f_1}{\mathrm{ch}^2\pi u}+\frac{f_2}{\mathrm{sh}^2\pi u}=1, \tag{5}$$

$$\frac{f_1}{\cos^2\pi v}-\frac{f_1}{\sin^2\pi v}=1, \tag{6}$$

where

$$f_1=\left(\frac{\mathrm{ch}\,\pi x\cos\pi y+\cos^2\frac{\pi}{2}\eta}{\sin^2\frac{\pi}{2}\eta}\right)^2;\quad f_2=\left(\frac{\mathrm{sh}\,\pi x\sin\pi y}{\sin^2\frac{\pi}{2}\eta}\right)^2.$$

Solving equation (5) for u and reverting to dimensional variables, we obtain the following expression for the temperature:

$$t=t_C\pm\frac{bq}{\pi\lambda_M}\,\mathrm{ar\,ch}\sqrt{\frac{f_1+f_2+1}{2}+\sqrt{\left(\frac{f_1+f_2+1}{2}\right)^2-f_1}}. \tag{7}$$

The signs (+) and (-) before the second term refer respectively to body 1 and body 2 (Fig. 1b).

Assuming now v = const in expression (6) will yield the equation for a family of thermal current lines. Solution of this equation for y and reversion to dimensional variables gives

$$Y=\frac{b}{\pi}\arccos\left\{-\frac{\mathrm{ch}\frac{\pi X}{b}\cos^2\left(\frac{\pi}{2}\frac{a}{b}\right)}{\mathrm{ch}^2\frac{\pi X}{b}+\frac{\mathrm{sh}^2\frac{\pi X}{b}}{\mathrm{tg}^2\pi v}}\pm\left[\mathrm{ch}^2\frac{\pi X}{b}\cos^4\left(\frac{\pi}{2}\frac{a}{b}\right)+\left[\frac{\mathrm{sh}^2\frac{\pi X}{b}}{\mathrm{tg}^2\pi v}+\sin^4\left(\frac{\pi}{2}\frac{a}{b}\right)\cos^2\pi v-\cos^4\left(\frac{\pi}{2}\frac{a}{b}\right)\right]\left(\mathrm{ch}^2\frac{\pi X}{b}+\frac{\mathrm{sh}^2\frac{\pi X}{b}}{\mathrm{tg}^2\pi v}\right)\right]^{1/2}\Bigg/\left(\mathrm{ch}^2\frac{\pi X}{b}+\frac{\mathrm{sh}^2\frac{\pi X}{b}}{\mathrm{tg}^2\pi v}\right)\right\}. \tag{8}$$

Solution of equation (5) for x with u = const gives (in dimensional variables) the isotherms equation in the following form:

$$X=\pm\frac{b}{\pi}\,\mathrm{ar\,ch}\left\{-\frac{\cos\frac{\pi Y}{b}\cos^2\left(\frac{\pi}{2}\frac{a}{b}\right)}{\cos^2\frac{\pi Y}{b}+\frac{\sin^2\frac{\pi Y}{b}}{\mathrm{th}^2\pi u}}+\left\{\mathrm{ch}^2\pi u\sin^4\left(\frac{\pi}{2}\frac{a}{b}\right)\left[\cos^2\frac{\pi Y}{b}+\frac{\sin^2\frac{\pi Y}{b}}{\mathrm{th}^2\pi u}\right]+\frac{\sin^2\frac{\pi Y}{b}}{\mathrm{th}^2\pi u}\left[\cos^2\frac{\pi Y}{b}+\frac{\sin^2\frac{\pi Y}{b}}{\mathrm{th}^2\pi u}-\cos^4\left(\frac{\pi}{2}\frac{a}{b}\right)\right]\right\}^{1/2}\Bigg/\left[\cos^2\frac{\pi Y}{b}+\frac{\sin^2\frac{\pi Y}{b}}{\mathrm{th}^2\pi u}\right]\right\}. \tag{9}$$

The orthogonal families of isotherms and thermal current lines are shown in Fig. 3 for two different values of parameter η.

By differentiating expression (7) with respect to X and Y it is possible to obtain equations for the temperature gradients $\partial t/\partial X|_{Y=\mathrm{const}}$ and $\partial t/\partial Y|_{X=\mathrm{const}}$. As an example, the temperature gradient $\partial t/\partial X|_{Y=b}$ as a function of the dimensionless distance parameter X/a is shown in Fig. 3 for different values of η with

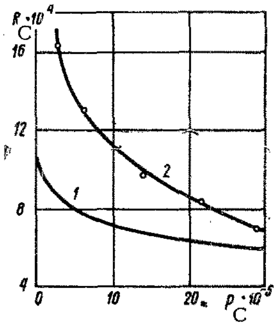

Fig. 4. Thermal contact resistance ($m^2 \cdot deg/W$) as a function of load (N/m^2) under vacuum at $t_C = 500\,°C$, for a VM-1/Kh18N9T contact pair; 1) theoretical calculations; 2) experimental results.

$q/\lambda_M = 1$. It is evident here that the temperature gradients in the contact zone increase sharply and attain their maximum values in the contact plane.

The quantity r_C in (3) will be defined as the difference

$$r_C = \Delta u(\infty) - \Delta u_1(\infty), \tag{10}$$

where

$$\Delta u(\infty) = u(\infty) - u(-\infty);$$
$$\Delta u_1(\infty) = u_1(\infty) - u_1(-\infty)$$

(the subscript 1 refers to an ideal contact, $\eta = 1$).

An analysis of equation (5) when $x = \pm\infty$ will show that function u asymptotically approaches the straight lines

$$u(\pm\infty) = x \mp \frac{2}{\pi} \ln \sin\left(\frac{\pi}{2}\eta\right). \tag{11}$$

With this taken into consideration, we obtain from expression (10)

$$r_C = -\frac{4}{\pi} \ln \sin\left(\frac{\pi}{2}\eta\right). \tag{12}$$

In dimensional variables the thermal contact resistance becomes

$$R_C = -\frac{4}{\pi}\frac{b}{\lambda_M} \ln \sin\left(\frac{\pi}{2}\frac{a}{b}\right). \tag{13}$$

In a general case, when the two bodies in contact have different thermal conductivities, the thermal contact resistance is

$$R_C = -\frac{4}{\pi}\frac{b}{\bar{\lambda}_M} \ln \sin\left(\frac{\pi}{2}\frac{a}{b}\right). \tag{14}$$

The magnitude of the relative contact area η can be determined from one of the formulas in [5]. Specifically, in many cases one may use the expression

$$\eta = \frac{p_C}{3\sigma_B}, \tag{15}$$

where σ_B is the ultimate strength of the weaker material in the contact pair.

Using (15) and taking into account (3), formula (14) can be rewritten as

$$R_C = -\frac{4}{\pi}\frac{b}{\bar{\lambda}_M} \ln \sin\left(\frac{\pi}{6}\frac{p_C}{\sigma_B}\right). \tag{16}$$

We go back now to expression (12) and note that it was derived assuming the bodies in contact to be infinitely long in the x-direction. The dimensions of the effective contact zone are very small, however, as long as essential changes in the temperature field are observed near the contact plane. For this reason, it is permissible in almost all practical cases to use the formulas derived here. It can be demonstrated that R_C quickly becomes independent of X. If the dimensions of bodies 1 and 2 (Fig. 1) along the x-axis equal $b/2$, for example, then R_C is practically the same as for infinitely long bodies [3].

The validity of formula (16) for calculating the thermal contact resistance of actual surfaces was verified by an experiment with a VM-1/Kh18N9T metal pair under vacuum (pressure of 10^{-4} mm Hg). The contact pressure was varied between $2.5 \cdot 10^5$ and $20 \cdot 10^5$ N/m^2, at a contact temperature of 500°C. The thermal contact resistance was determined according to the formula

$$R_C = \frac{\Delta t_C}{q}, \tag{17}$$

where Δt_C is the temperature drop across the contact found by linear extrapolation of specimen temperatures up to the contact plane.

The maximum error in this determination of R_C amounted to about 10%.

The quality of the surface finish on the contact pair was checked with a profilograph-profilometer of the Kalibr Works. The VM-1 alloy specimen had been ground to a Class 10 surface fineness, while the Kh18N9T steel specimen had a Class 4 surface fineness with a mean microroughness of 24 μm. The radius of grinding tracks was about 10^3 times the width $2a$ of the contact zone and, consequently, the effect of the curvature of the grinding track could be disregarded assuming the track to be straight.

In examining the contact surface of the Kh18N9T steel specimen on the profilograph-profilometer we found a waviness (in the direction perpendicular to the grinding tracks) with a step of $5\text{-}6 \cdot 10^{-3}$ m. The crest height of this waviness was approximately the same as that of the microroughness. These waviness parameters agree approximately with the results shown in the picture collection of surface quality attained by different surface treatments [4].

As a result of an existing waviness condition, an actual contact between surfaces is formed only by the microprotrusions at the wave peaks.

It is evident from Fig. 4 that the discrepancy between calculated ($b = 3 \cdot 10^{-3}$ m) and experimental data is greater at lighter loads than at heavier loads. This can obviously be explained by the fact that the heights of the wave peaks vary somewhat. Thus, it is noted in [6] that the difference between the heights of individual waves can amount to about 20%. When the relative contact area η is small, therefore, the load is taken up by the highest microprotrusions only and, consequently, the distance 2b between adjacent microprotrusions within the contact region may be greater than the waviness pitch.

The change in dimension b due to loading can be accounted for, if the wave shape as well as the shape and the height distribution of microprotrusions are known.

In this way, the thermal contact resistance of a planar contact under vacuum can, to the first approximation, be calculated by formula (16). For a more accurate calculation, it is necessary to account for the change in dimension b due to loading.

NOTATION

X, Y	are the coordinates (Fig. 1);
$2a$	is the width of contact zone;
2b	is the contact pitch;
η	is the relative contact area;
h	is the difference between ideal and actual relative contact areas;
x, y	are the dimensionless coordinates;
t	is the temperature;
t_C	is the contact temperature;
q	is the heat flux;
p_C	is the contact pressure;
σ_B	is the ultimate strength;
λ_M	is the thermal conductivity;
$\bar{\lambda}_M = 2\lambda_{M_1}\lambda_{M_2}/(\lambda_{M_1} + \lambda_{M_2})$	is the equivalent thermal conductivity (bodies 1 and 2);
u, v	are the conjugate harmonic functions (u is the temperature function, v is the thermal current function);
u_1	is the u-function for an ideal contact ($\eta = 1$);
$u(\infty)$	is the u-function at $x = \infty$;
$u(-\infty)$	is the u-function at $x = -\infty$;
R_C	is the thermal contact resistance;
r_C	is the relative thermal contact resistance.

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